Topic 15 CLT Example Problem

15-7 $\mu = 850\$$

$\sigma = 250\$$

X = amount expected to spend on X-mas presents

a) If $X \sim N(\mu, \sigma)$ then $n=5$ is okay (Normal Dist.)

If X is not $N(\mu,\sigma)$, then $n=5$ is insufficient. At least 30 is advisable

b) yes. By the CLT, The sampling distribution of the sample mean is Normal.

$\bar{X} \sim N(\mu, \sigma/\sqrt{n})$

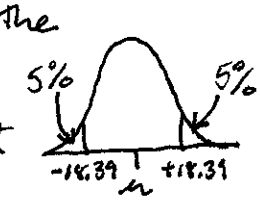
c) use the z-score (since $\bar{x}$ is normal, this can be done)

$$Z_u = \frac{x_i - \mu}{\sigma/\sqrt{n}} = \frac{18.39}{250/\sqrt{500}} = \frac{18.39}{250}\sqrt{500} = 1.64$$

$$\Phi(1.645) \sim .95$$

So $\pm \$18.39$ corresponds to 90% probability of the sample mean falling in that interval.

(Because $1-.95=.05$ on the upper portion, but there is also .05 on the lower portion, and $.05+.05=.1$, which is associated with $1-.1=.9$ or 90% probability. see diagram)

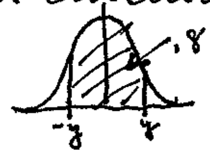


d) same as c), but $Z_u = \frac{21.91}{250/\sqrt{500}} = 1.95$ ....

e) same as as d & c

f) Same ~~calculate~~ equation, but calculate in reverse.

$$\Phi_H(y) - \Phi_L(-y) = .8$$



By symmetry $\Phi_H(y) - \Phi_0(0) = .4$

so $\Phi_H(y) = .4 + \Phi_0(0) = .4 + .5 = .9$ $\quad y \sim 1.28$ by the table T-4

then $\frac{K}{250}\sqrt{500} = 1.28$

$K = (250)(1.28)/\sqrt{500} = 14.31$

$\boxed{K = 14.31}$